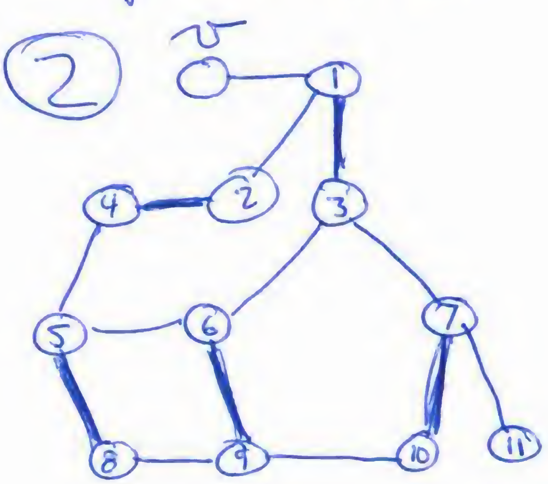
(1) $S^+ = \{2,4,4,2,2,6,2,8,3\}$

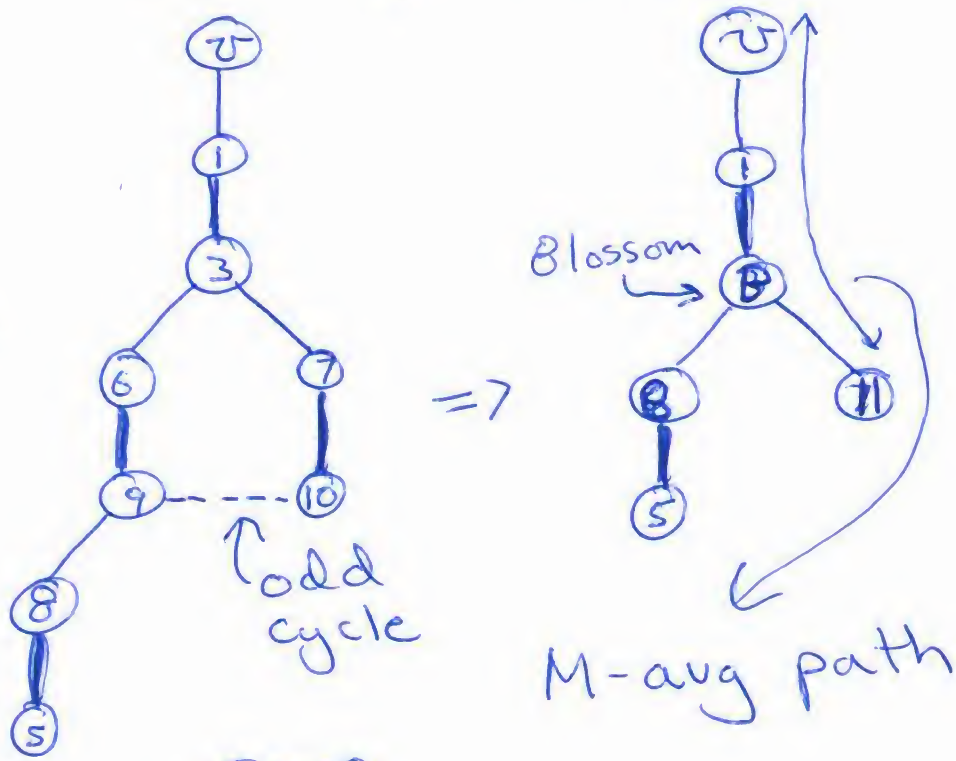
$S^- = \{3,2,6,2,2,4,4,2,8\}$

We note that there exists the same set of integers in the out-degree set S^+ and the in-degree set S^-. A Euler Tour would exist if the graph is connected and for each vertex v $d^+(v) = d^-(v)$. So for S^+, S^- it is possible.

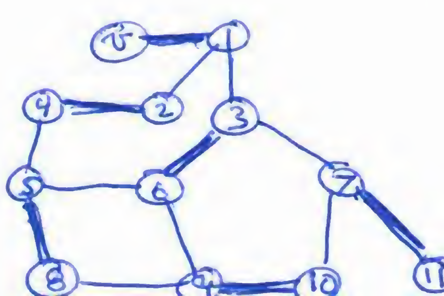
(2)



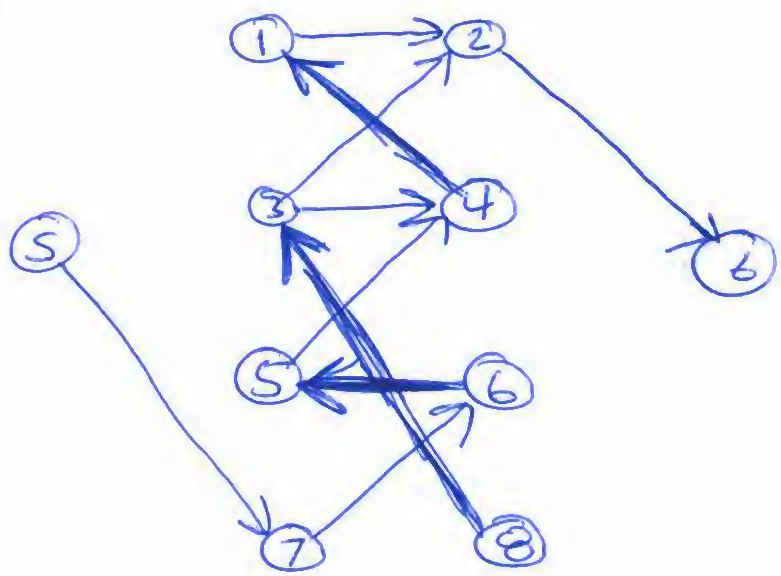
Starting from v

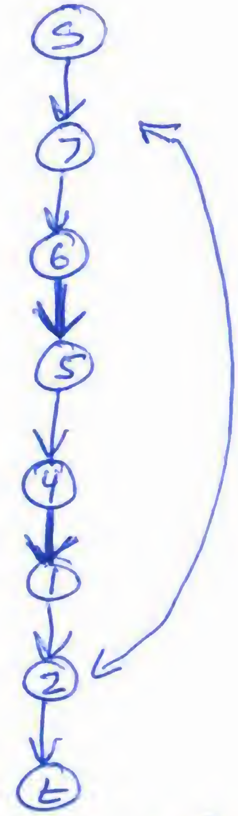


New match

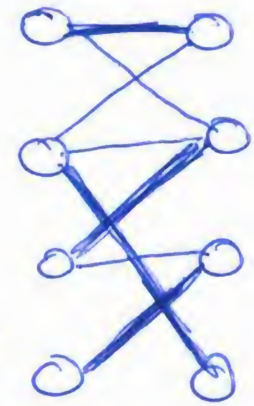


3



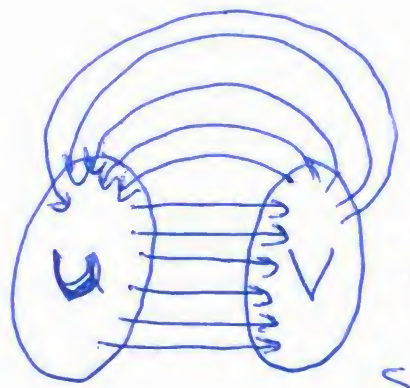


M-aug path



New match

4



Note that regardless of how we define the vertex sets defining the cut, we are interested in finding some minimum. In the above, we see the minimum cut between the two sets is defined by $[V, U]$, $|[V,U]|=5$

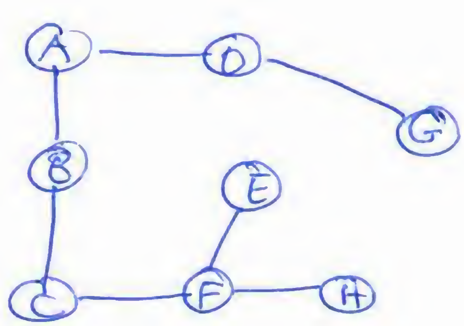
Knowing nothing else, we can place our bound as $\boxed{K'(G) \le 5}$

5 Dijkstra's

Vert processed	distances A	B	C	D	E	F	G	H
—	0	∞	∞	∞	∞	∞	∞	∞
A	0	3	∞	1	∞	∞	∞	∞
D	0	3	∞	1	5	∞	4	∞
B	0	3	6	1	5	7	4	∞
G	0	3	6	1	5	7	4	8
E	0	3	6	1	5	6	4	8
C	0	3	6	1	5	6	4	8
F	0	3	6	1	5	6	4	8
H	0	3	6	1	5	6	4	8

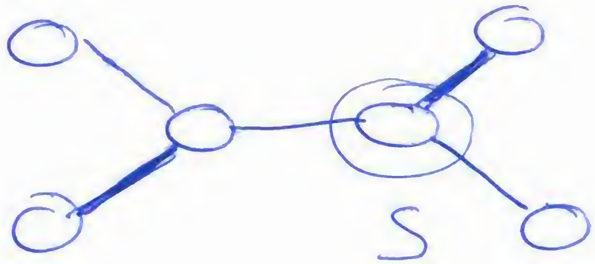
Krushkal's

Edges added: (A, D), (E, F), (F, H), (C, F), (B, C), (D, G), (A, B)



Note: tree is not necessarily unique

6



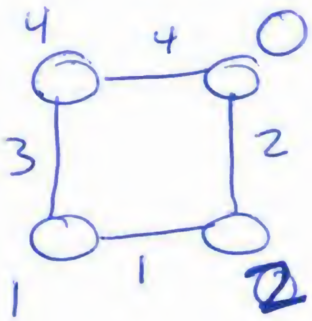
$|V(G)|=6$

$|M|=2$

$o(G-S)=3>|S|=1$

A perfect match would require $|M'|=3$

7



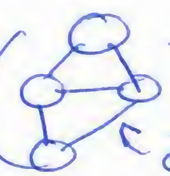
Note: absolute difference in vertex labels is unique for each edge

8 From König-Egervary, the size ~~of~~ a min cover = size of a max match. As $|C|=7$, we can also place an equality on the size of a max match M' as $\boxed{|M'|=|C|=7}$

⑨ As Hall's condition doesn't hold, we know there is no perfect match saturating X. Hence, the current match M has $|M| = 8$ and is therefore maximum. Using König-Egervary again, we can place a tight "bound" on the size of a min cover C to be

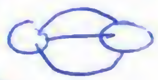
$$\boxed{|C| = |M| = 8}$$

⑩ $\tau(G) = \tau(G \cdot e) + \tau(G - e)$

$\tau\Big($  $\Big) = \tau\Big($  $\Big) + \tau\Big($  $\Big)$ ← C_3 + a tree = 3 spanning trees

$= \tau\Big($  $\Big) + \Big($  $\Big) + 3$

3 possible edges = 3 C_2 + tree = 2

$= \tau\Big($  $\Big) + \tau\Big($  $\Big) + 3 + 3$

$= 3 + 2 + 3 + 3 = \boxed{11}$

(11) We know $K(G) \leq K'(G) \leq \delta(G)$

As given, we have $\delta(G) = 4$

From class, we proved that if

$\forall u, v \in V(G) : u, v \in C$ then G

is 2-connected, or $K(G) = 2$

so $\boxed{2 \leq K'(G) \leq 4}$

(12) $S = \{d_1 \dots d_n\}$ realizeds a tree

iff $\forall i : d_i \geq 1$ and $\sum_{i=1}^{n} d_i = (2n-2)$

if $S = \{d_1 \dots d_n\}$ realizes a tree

$\Rightarrow \forall i : d_i \geq 1$ and $\sum_{i=1}^{n} d_i = (2n-2)$

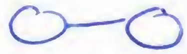
- We know a tree is connected, so all $d_i > 0$ ✓

- We know a tree on n vertices will have $n-1$ edges. By our degree sum formula $\sum_{i=1}^{n} d_i = 2m = 2(n-1) = (2n-2)$ ✓

If $\forall i: d_i \geq 1$ and $\sum_{i=1}^{n} d_i = (2n-2)$ 12 cont

$\Rightarrow$ degree sequence realizes a tree

- We'll use induction on vertices

Base:  both $d_i > 0$ $\sum d_i = 2 = (4-2)$ ✓

I.H.: We assume for some $P(k)$ that our degree sequence realizes a tree

I.S.: Let's consider $P(n) = P(k+1)$ where obviously $n > k$, $k = n-1$

- We note by our degree sum formula, that there must be at least one vertex of degree 1, as obviously $(2n-2) < 2n \Leftarrow$ if all $d_i = 2$

- We remove that vertex, and note that it subtracts its degree of 1 and 1 from its neighbor from the sum

$P(n) \Rightarrow \sum^{n} d_i = 2n-2 \Rightarrow 2n-2-2 = 2n-4$

$P(n-1) \Rightarrow \sum^{n-1} d_i = 2(n-1)-2 = 2n-4$ ✓

- We show our assumption holds, we invoke our I.H. on $P[k] = P(n-1)$, we note adding a leaf to a tree won't create a cycle $\Rightarrow$ our $P(n)$ case is a tree $\square$